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BRIEF COMMUNICATION

DISPERSION OF A SOLUTE IN A MICROPOLAR PIPE FLOW

GOODARZ AHMADI
Pahlavi University, Shiraz, Iran

1. INTRODUCTION

Taylor (1953, 1954a, b) discussed the dispersion of soluble matter in the viscous, incompressible laminar flow of a fluid in a circular pipe. The case of the dispersion of a solute in non-Newtonian fluid flow in a circular pipe was discussed by Fan & Hwang (1965). Recently Soundalgekar (1971) studied the effect of couple stresses on the dispersion of a solute in a channel flow.

In the present work the dispersion of a soluble matter in a micropolar pipe flow is considered and the effective Taylor diffusivity is calculated. The effects of various dimensionless parameters on the effective diffusivity are discussed.

2. BASIC EQUATIONS

The expression for the velocity in a fully developed pipe flow as derived by Eringen (1966), is given by

$$\frac{u}{u_o} = 1 - \rho^2 + \frac{k}{\mu + k}\frac{1}{\lambda}\frac{I_o(\lambda)}{I_1(\lambda)}\left[\frac{I_o(\lambda\rho)}{I_o(\lambda)} - 1\right], \qquad [1]$$

where

$$u_o = \frac{-R}{2(2\mu + k)}\frac{\mathrm{d}p}{\mathrm{d}x}, \quad \lambda = \left(\frac{2\mu + k}{\mu + k}\frac{k}{\gamma}\right)^{1/2} R, \quad \rho = \frac{r}{R}. \qquad [2]$$

Here u_o is the maximum velocity, R is the radius of the pipe and $\mathrm{d}p/\mathrm{d}x$ is pressure gradient along the pipe. μ, k and γ are the coefficients of viscosity. $I_o(\,)$ and $I_1(\,)$ are the modified Bessell functions of the zeroth and first order, respectively. Also note that the dimension of k is the same as μ but the dimension of γ is viscosity times length square. Hence λ is a dimensionless quantity.

The concentration c of the solute diffusing in the fluid satisfies the following equations

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\left[\frac{\partial^2 c}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right)\right], \qquad [3]$$

where D is the molecular diffusion coefficient.

Following Taylor (1953, 1954a, b) we assume that the axial diffusion is much smaller than the radial diffusion.

Using the dimensionless quantities

$$\xi = (x - \bar{u}t)/L, \quad \theta = tL/\bar{u}, \tag{4}$$

the diffusion equation [3] in a frame moving with the average velocity $\bar{u}$ becomes

$$\frac{\bar{u}}{L}\frac{\partial c}{\partial \theta} + \frac{w}{L}\frac{\partial c}{\partial \xi} = \frac{D}{R^2}\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial c}{\partial \rho}\right). \tag{5}$$

where

$$\begin{aligned} w/u_o &= (u - \bar{u})u_o \\ &= \frac{1}{2} - \rho^2 + \frac{k}{\mu + k}\frac{1}{\lambda}\frac{1}{I_1(\lambda)}\left[I_o(\lambda\rho) - \frac{2}{\lambda}I_1(\lambda)\right], \end{aligned} \tag{6}$$

and where L is a given length along the flow direction.

Assume Taylor's limiting conditions to be applicable, then the partial equilibrium may be assumed in any cross section of the pipe and c then satisfies:

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial c}{\partial \rho}\right) = \frac{R^2}{DL}w\frac{\partial c}{\partial \xi}. \tag{7}$$

Substituting for w from [6], multiplying by ρ, integrating once, then dividing by ρ and integrating again we find

$$c = \frac{R^2u_o}{DL}\left\{\frac{\rho^2}{8} - \frac{\rho^4}{16} + \frac{k}{\mu + k}\frac{1}{\lambda^2}\frac{1}{I_1(\lambda)}\left[\frac{1}{\lambda}I_o(\lambda\rho) - \frac{\rho^2}{2}I_1(\lambda)\right]\right\}\frac{\partial c}{\partial \xi} + c_o, \tag{8}$$

where c satisfies the boundary condition

$$\left.\frac{\partial c}{\partial \rho}\right|_{\rho = 1} = o, \tag{9}$$

and c_o is a constant which can be determined from the entry condition.

Now the volume rate of the transport of the solute across a section of the pipe is given by

$$Q = \int_o^R 2\pi r\, cw\, \mathrm{d}r. \tag{10}$$

Inserting for c and w

$$Q = \frac{2\pi R^4 u_o^2}{DL}F\left(\frac{k}{\mu}, \lambda\right)\frac{\partial c}{\partial \xi}, \tag{11}$$

Table 1. Values of $F(k/\mu, \lambda) \times 10^{+3}$.

K/μ \ λ	1	2	3	4
0	2.20	2.20	2.20	2.20
1	86.49	3.79	1.40	0.90
2	143.40	4.43	1.11	0.59
3	177.26	4.77	0.96	0.39
4	199.30	4.98	0.87	0.27
5	214.72	5.13	0.80	0.19
6	226.08	5.23	0.76	0.13
7	234.80	5.31	0.73	0.09
8	241.69	5.37	0.70	0.06
9	247.28	5.42	0.68	0.04

where

$$F\left(\frac{k}{\mu}, \lambda\right) = \int_o^1 \rho \left\{\frac{1}{2} - \rho^2 + \frac{k}{\mu + k}\frac{1}{\lambda}\frac{1}{I_1(\lambda)}\left[I_o(\lambda\rho) - \frac{2}{\lambda}I_1(\lambda)\right]\right\}$$
$$\left\{\frac{\rho^2}{8} - \frac{\rho^4}{16} + \frac{k}{\mu + k}\frac{1}{\lambda^2}\frac{1}{I_1(\lambda)}\left[\frac{1}{\lambda}I_o(\lambda\rho) - \frac{\rho^2}{2}I_1(\lambda)\right]\right\}\mathrm{d}\rho. \quad [12]$$

On comparing [11] with Fick's law of diffusion, one can show that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective Taylor diffusion coefficient D^* given by

$$D^* = \frac{2R^2u_o^2}{D} F\left(\frac{k}{\mu}, \lambda\right). \quad [13]$$

The numerical values of $F(k/\mu, \lambda)$ are given in table 1.

3. CONCLUSION

The effective Taylor diffusion coefficient is found to be governed by two nondimensional parameters k/μ and λ. For $\lambda = 1$, $F(k/\mu, \lambda)$ increases rapidly with k/μ. The rate of increase is slower for $\lambda = 2$. However, for $\lambda = 3$ the effective diffusivity decreases with an increase in k/μ. For large values of λ and k/μ (>30), $F(k/\mu, \lambda)$ remains approximately constant ($=2.2 \times 10^{-3}$). In this range the effective diffusivity becomes independent of λ and k/μ and approaches that of a simple viscous fluid.

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